Errata for

Algebra in the Stone-Čech Compactification, 2nd Edition,
DeGruyter, Berlin, 2012

by

Neil Hindman and **Dona Strauss**

Typos

Included here are the typographical errors of which the authors are aware at the time of this writing.

Page 132, line -2. "$\widetilde{\theta}(m) \neq \widetilde{\theta}(n)$" should be "$\widetilde{\theta}(q_m) \neq \widetilde{\theta}(q_n)$".

Page 133, line 5. "$\langle i_1, i_2, \ldots, i_n \rangle$" should be "$\langle i_1, i_2, \ldots, i_k \rangle$"

Page 136, Corollary 6.23. "$e, f \in E(S)$" should be "$e, f \in E(\beta S)$".

Page 224, line -8. "set of sums of all P-sums" should be "set of all P-sums".

Page 345, line 20. "$L = (\{1,2\},\{4,5,6\},\{9,10,11,12\},\{14,15\})$" should be "$L = (\{1,2\},\{4,5,6\},\{10,11,12,13\},\{15,16\})$"

Page 374, Definition 15.16.1(a). "$\vec{y} \in \mathbb{Q}^v$" should be "$\vec{y} \in \mathbb{Q}^u$".

Section 8.5

Lemma 8.48 is incorrect as stated. Following is a corrected version with its proof.

Lemma 8.48. *Let $a \in G \setminus \{e\}$ and pick $r \in \mathbb{N}$ such that $a < b_r$. Let $k, l \in \mathbb{N}$ and assume that $b_{m_1} b_{m_2} \cdots b_{m_k}$ and $b_{n_1} b_{n_2} \cdots b_{n_l}$ are P-products such that $a b_{m_1} b_{m_2} \cdots b_{m_k} = b_{n_1} b_{n_2} \cdots b_{n_l}$ and $b_{m_1} \in P_r$. Then $k < l$ and, if $i = l - k$, then $a = b_{n_1} b_{n_2} \cdots b_{n_i}$ and $m_j = n_{i+j}$ for each $j \in \{1, 2, \ldots, k\}$.*

Proof. Suppose the conclusion fails and pick a counterexample with $k + l$ a minimum among all counterexamples. Note that $b_{m_1} \in P_r$ and $\{b_1, b_2, \ldots, b_r\} \subseteq F_r$ so $m_1 > r$.

Assume first that $k > 1$ and $l > 1$. If $n_{l-1} \geq m_{k-1}$, we have

$$b_{n_l} = (b_{n_1} \cdots b_{n_{l-1}})^{-1} (a b_{m_1} \cdots b_{m_{k-1}}) b_{m_k}$$

and, since $b_{n_l} \notin F_{n_{l-1}} P$, we must have that $a b_{m_1} \cdots b_{m_{k-1}} = b_{n_1} \cdots b_{n_{l-1}}$ and $b_{n_l} = b_{m_k}$ so there is a smaller counterexample. Similarly, if $n_{l-1} < m_{k-1}$, we get a smaller counterexample because of the equation $b_{m_k} = (a b_{m_1} \cdots b_{m_{k-1}})^{-1} (b_{n_1} \cdots b_{n_{l-1}}) b_{n_l}$.

Therefore we must have $k = 1$ or $l = 1$. Suppose that $l = 1$. Then $b_{m_1} \cdots b_{m_k} = a^{-1} b_{n_1}$. If $k = 1$, this says that $b_{m_1} = a^{-1} b_{n_1} \in F_r P$, a contradiction. If $k > 1$, this says $b_{m_k} = (ab_{m_1} \cdots b_{m_{k-1}})^{-1} b_{n_1}$ so $b_{m_k} \in F_{m_{k-1}} P$ unless $ab_{m_1} \cdots b_{m_{k-1}} = e$. But if $k-1 = 1$, the equation $ab_{m_1} = e$ says that $b_{m_1} \in F_r$ while if $k-1 > 1$, the equation $ab_{m_1} \cdots b_{m_{k-1}} = e$ says that $b_{m_{k-1}} \in F_{m_{k-2}}$.

Thus we must have $k = 1$ and $l > 1$. If $n_{l-1} \leq r$ we get $b_{m_1} = a^{-1}(b_{n_1} \cdots b_{n_{l-1}}) b_{n_l}$ so $a^{-1}(b_{n_1} \cdots b_{n_{l-1}}) = e$; that is $a = b_{n_1} \cdots b_{n_{l-1}}$ and $m_1 = n_l$, so this is not a counterexample. If $n_{l-1} > r$, we get $b_{n_l} = (b_{n_1} \cdots b_{n_{l-1}})^{-1} a b_{m_1}$ so $(b_{n_1} \cdots b_{n_{l-1}})^{-1} a = e$ and we again conclude that we don't have a counterexample. □

The lemmas and the theorem that cited Lemma 8.48 are all correct as stated (except for a typo in the statement of Lemma 8.64), but all need adjustments to their proofs – in the case of Lemma 8.49, the proof needs replacement.

Lemma 8.49. *The expression for an element of T as a P-product is unique.*

Proof. Assume that there are P-products $b_{m_1} b_{m_2} \cdots b_{m_k}$ and $b_{n_1} b_{n_2} \cdots b_{n_l}$ such that $b_{m_1} b_{m_2} \cdots b_{m_k} = b_{n_1} b_{n_2} \cdots b_{n_l}$ but $(m_1, m_2, \ldots, m_k) \neq (n_1, n_2, \ldots, n_l)$ and pick such products with $k + l$ a minimum. As in the proof of Lemma 8.48 above, if $k > 1$ and $l > 1$, then $b_{n_k} = b_{n_l}$ and so the equation $b_{m_1} b_{m_2} \cdots b_{m_{k-1}} = b_{n_1} b_{n_2} \cdots b_{n_{l-1}}$ provides a smaller example.

Thus we can assume without loss of generality that $k = 1$. If also $l = 1$, then $b_{m_1} = b_{n_1}$, so we must have $l > 1$. But then $(b_{n_1} \cdots b_{n_{l-1}})^{-1} b_{m_1} = b_{n_l}$ and so $b_{n_l} \in F_{n_{l-1}} P$, a contradiction. □

For the proof of Lemma 8.57, the sentence "For each $a \in G$, the set $X_a = \{b_n : n \in Q\,,\, b_n > a, \text{ and } b_n > a^{-1}\} \in x$." should be replaced by "For each $a \in G$, pick $r_a \in \mathbb{N}$ such that $a < b_{r_a}$ and let $X_a = \{b_n : n \in Q \text{ and } b_n \in P_{r_a}\}$. Note that $X_a \in x$."

For the proof of Lemma 8.59, the sentence "For each $a \in G$, let Q_a denote the set of P-products $b_{n_1} b_{n_2} \cdots b_{n_k}$ with $b_{n_1} > a$ and $b_{n_1} > a^{-1}$." should be replaced by "For each $a \in G$, pick $r_a \in \mathbb{N}$ such that $a < b_{r_a}$ and let Q_a denote the set of P-products $b_{n_1} b_{n_2} \cdots b_{n_k}$ such that $b_{n_1} \in P_{r_a}$."

For the proof of Theorem 8.63, the sentence "If we choose n such that $a < b_n$ and $a < b_n^{-1}$, it follows from Lemma 8.48 that $aT_n \cap T_m = \emptyset$." should be replaced by "If we choose $r \in \mathbb{N}$ such that $a < b_r$ and choose n such that $b_n \in P_r$, it follows from Lemma 8.48 that $aT_n \cap T_m = \emptyset$."

Finally, the statement of Lemma 8.64 needs to specify that $x \neq e$ and the proof needs revision.

Lemma 8.64. *Let G be a countably infinite discrete group and let p be a right cancelable element of G^*. Suppose that $x \in \beta G \setminus \{e\}$, $y \in T_\infty$, and $xy \in \overline{T}$. Then $x \in \overline{T}$.*

Proof. Suppose that $x \notin \overline{T}$ and let $X = G \setminus (T \cup \{e\})$. For each $a \in X$, pick $r_a \in \mathbb{N}$ such that $a < r_a$. Let Z be the set of all products of the form $ab_{n_1}b_{n_2}\cdots b_{n_k}$ where $b_{n_1}b_{n_2}\cdots b_{n_k}$ is a P-product, $a \in X$, and $b_{n_1} \in P_{r_a}$. By Theorem 4.15, $Z \in xy$. Since $T \in xy$, pick $a \in X$ and a P-product $b_{n_1}b_{n_2}\cdots b_{n_k}$ such that $b_{n_1} \in P_{r_a}$ and $ab_{n_1}b_{n_2}\cdots b_{n_k} \in T$. Then by Lemma 8.48, $a \in T$, a contradiction. $\square$

Section 14.4

In the proof of Theorem 14.14.4, we neglected to show that $J(S) \neq \emptyset$. This is most easily accomplished by moving Theorem 14.14.4 to after Lemma 14.14.6 and then invoking Theorem 3.11.