

Patterns in Mathematics (012-01)
Final Exam
Spring 2015

Please do all work on this paper. Each problem is worth 20 points. Answers that require computation will get no credit if the computation is not shown.

1. Identify each of the following as a negation, disjunction, conjunction, or conditional. (A conditional statement is also called an implication.)

(a) $\sim P \rightarrow Q \vee R$

(b) $\sim (P \rightarrow Q) \vee R$

(c) $\sim (P \rightarrow Q \vee R)$

(d) $(\sim P \rightarrow Q) \vee R$

2. Let

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 3, 5, 7, 9\}$$

$$C = \{2, 4, 6, 8\}$$

(a) $A \cap B =$

(b) $A \cup C =$

(c) $(A \cap B) \times C =$

(d) $(A \cup C) \cap B =$

3. A jury of six people is to be chosen at random from a group of ten men and nine women. Find the probability that the jury will consist of three men and three women.

4. A pair of dice, one red and one green, are tossed.

(a) List the elements of the sample space.

(b) What is the probability the number on the red die is larger than the number on the green die?

(c) What is the probability that the numbers on both dice are equal?

5. $1,719,900 = 2^2 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 13$ and $267,750 = 2 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 17$. You may write the solutions to the following two problems as a product of primes.

(a) Find the least common multiple of 1,719,900 and 267,750.

(b) Find the greatest common divisor of 1,719,900 and 267,750.

6. Fill in the following truth tables. In each case, circle the column with the final truth values for the given statement.

| P | Q | $\sim (P \wedge \sim Q)$ |
|-----|-----|--------------------------|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

| P | Q | R | $Q \rightarrow (P \vee R)$ |
|-----|-----|-----|----------------------------|
| T | T | T | |
| T | T | F | |
| T | F | T | |
| T | F | F | |
| F | T | T | |
| F | T | F | |
| F | F | T | |
| F | F | F | |

7. Perform the indicated operations in base five. (No credit will be given for converting to base ten, doing the computation, and converting back to base five.)

(a)
$$\begin{array}{r} 214_{\text{five}} \\ + 102_{\text{five}} \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 214_{\text{five}} \\ \times 102_{\text{five}} \\ \hline \end{array}$$

8. An urn contains 3 red balls, two blue balls, and one green ball. Two balls are removed without replacement.

(a) What is the probability that at least one of them is green?

(b) What is the probability that both of them are green?

(c) What is the probability that neither of them is red?

9. Write the following as a quotient of two integers. (You do not need to reduce to lowest terms.)

(a) $21.\overline{237}$

(b) $3.\overline{23}$

10. Find the prime factorization of 6552.