

Calculus II (157–01)
First Exam
Spring 2014

Please do all work on this paper. Points are written to the left of each problem,

- 28 pts 1. (For this problem, you may use the method of your choice for each part, but you will probably find it convenient to use the disk method for part (a) and the shell method for part (b).)

Find the volume of the solid generated when the region in the first quadrant bounded by the graphs of $y = \sqrt{x^2 + 1}$, $x = 0$, $x = 1$, and $y = 0$ is revolved

(a) around the x -axis.

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(b) around the y -axis.

- 10 pts 2. Find the general solution to the differential equation $\frac{dy}{dx} = x^2y^3$. You may leave your answer in terms of both x and y .

14 pts 3. Do one of problems (a) or (b). (If you do both, they will be graded at 7 points each.)

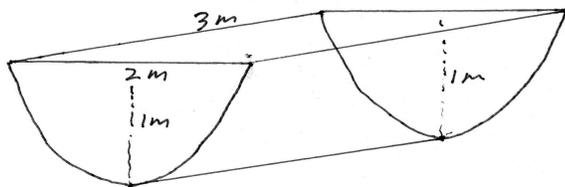
(a) Find the arc length of the graph of $f(x) = (x - 4/9)^{3/2}$ for $1 \leq x \leq 4$.

(b) Find the surface area generated when the graph of $f(x) = \sqrt{x}$ for $1 \leq x \leq 4$ is revolved around the x -axis.

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10 pts 4. A group of animals had a population of 1000 in the year 2000 and a population of 1300 in the year 2010. What is the expected population in the year 2015 assuming that the birth and death rates are constant over the entire period?

- 28 pts 5. A trough has a face in the shape of the parabola $y = x^2$. It is 2 meters across at the top, 3 meters long, and 1 meter deep. The trough is full of water which has a weight density of $\delta = 9810$ Newtons/meter³.



- (a) Write an integral whose value is the force on one of the faces which is in the shape of a parabola.

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- (b) Write an integral whose value is the work needed to pump the water out over the top edge of the trough.

- 10 pts 6. Find the centroid of the region in the first quadrant bounded by the coordinate axes and the circle $x^2 + y^2 = 1$. (You are allowed to think. You know the area of one fourth of a circle of radius 1 is $\frac{\pi}{4}$ and you can see that $\bar{x} = \bar{y}$.)