

Calculus II (157–01)

Third Exam

Spring 2016

Please do all work on this paper. Points are written to the left of each problem.
No calculators are allowed and no cell phones are allowed.

14 pts 1. Let $f(x) = 1 + 6x^{2/3}$. Find the length of the graph of f from $x = 1$ to $x = 8$.

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8 pts 2. Find all polar coordinates $\langle r, \theta \rangle$ with $0 \leq \theta < 2\pi$ for the point with Cartesian coordinates $(-\sqrt{3}, 1)$.

11 pts 3. Find a Cartesian equation for the graph of the polar equation $r = r \tan \theta$.

14 pts 4. Find the surface area of the region generated when the graph of $r = 2 \sin \theta$ from $\theta = \frac{\pi}{6}$ to $\theta = \frac{\pi}{3}$ is revolved around the x -axis.

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20 pts 5. A sequence $\langle a_n \rangle_{n=1}^{\infty}$ is defined recursively by

$$\begin{aligned} a_1 &= 2 \text{ and} \\ a_{n+1} &= \left(-\frac{2}{3}\right)a_n. \end{aligned}$$

(a) Find the first three terms of $\langle a_n \rangle_{n=1}^{\infty}$.

(b) Does the series $\sum_{n=1}^{\infty} a_n$ converge? If so, find its sum.

14 pts 6. A sequence $\langle a_n \rangle_{n=1}^{\infty}$ has the property that

- (i) $a_1 = 2$;
- (ii) for each n , $a_n < a_{n+1}$; and
- (iii) for each n , $a_n < 5$.

What can you say about $\lim_{n \rightarrow \infty} a_n$?

22 pts 7. Determine, giving reasons, whether the following series converge.

(a) $\sum_{n=1}^{\infty} \frac{n^2 + 1}{50n^2 + 10n}$.

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(b) $\sum_{k=1}^{\infty} \frac{(-3)^k}{4^{k+1}}$.